Additional Mathematics (O and N(A)-Level) Teaching and Learning Syllabus

The document contains general information about the secondary mathematics curriculum and content for the O- and N(A)-Level Additional Mathematics syllabuses. The syllabuses will be implemented at Secondary 3 and 4 in 2013 and 2014 respectively.

Ministry of Education
SINGAPORE

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Learning Mathematics

A 21st Century Necessity

Learning mathematics is a key fundamental in every education system that aims to prepare its citizens for a productive life in the 21st century.

As a nation, the development of a highly-skilled and well-educated manpower is critical to support an innovation- and technology-driven economy. A strong grounding in mathematics and a talent pool in mathematics are essential to support the wide range of value-added economic activities and innovations. Many countries are paying attention to the quality of their mathematics education. The growing interest in TIMSS and PISA speaks of the global interest and importance placed on mathematics education.

At the individual level, mathematics underpins many aspects of our everyday activities, from making sense of information in the newspaper to making informed decisions about personal finances. It supports learning in many fields of study, whether it is in the sciences or in business. A good understanding of basic mathematics is essential wherever calculations, measurements, graphical interpretations and statistical analysis are necessary. The learning of mathematics also provides an excellent vehicle to train the mind, and to develop the capacity to think logically, abstractly, critically and creatively. These are important 21st century competencies that we must imbue in our students, so that they can lead a productive life and be life-long learners.

Students have different starting points. Not all will have the same interests and natural abilities to learn mathematics. Some will find it enjoyable; others will find it challenging. Some will find the theorems and results intriguing; others will find the formulae and rules bewildering. It is therefore important for the mathematics curriculum to provide differentiated pathways and choices to support every learner in order to maximise their potential. The curriculum must engage the 21st century learners, who are digital natives comfortable with the use of technologies and who work and think differently. The learning of mathematics must take into cognisance the new generation of learners, the innovations in pedagogies as well as the affordances of technologies.

It is the goal of the national mathematics curriculum to ensure that all students will achieve a level of mastery of mathematics that will serve them well in their life, and for those who have the interest and ability, to pursue mathematics at the highest possible level. Mathematics is an important subject in our national curriculum. Students begin to learn mathematics from the day they start formal schooling, and minimally up to the end of secondary education. This gives every child at least 10 years of meaningful mathematics education.

About this document

This document provides an overview of the curriculum. It explains the design of the curriculum from the primary to the pre-university level, and provides details of the O and N(A)-Level Additional Mathematics syllabuses, including the aims, content, outcomes and the approach to teaching and learning.

This document comprises 5 chapters as described below.

Chapter 1 provides an overview of the curriculum review, the goals and aims of the different syllabuses of the entire mathematics curriculum (primary to pre-university) as well as the syllabus design considerations across the levels.

Chapter 2 elaborates on the Mathematics Framework which centres around mathematical problem solving. The framework serves as a guide for mathematics teaching, learning and assessment across the levels.

Chapter 3 focuses on the process of teaching and learning so as to bring about engaged learning in mathematics. It highlights the principles of teaching and phases of learning as well as the learning experiences to influence the way teachers teach and students learn so that the aims of the curriculum can be met. The role of assessment and how it can be integrated to support learning in the classroom is also highlighted in this chapter.

Chapter 4 details the O-Level Additional Mathematics syllabus in terms of its aims, syllabus organisation, mathematical processes, content and learning experiences.

Chapter 5 details the N(A)-Level Additional Mathematics syllabus in terms of its aims, syllabus organisation, mathematical processes, content and learning experiences.

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Background

Staying Relevant and Forward-Looking

As in all previous reviews, the 2010 full-term review aims to update the syllabuses so that they continue to meet the needs of our students, build a strong foundation in mathematics, and make improvement in the school mathematics education. It takes into consideration the analyses of students' performances in national examinations as well as international studies such as TIMSS and PISA. This review also takes on board the curriculum-wide recommendations from envisaging studies into the overall Singapore curriculum such as seeking a better balance between content and skills, creating opportunities to develop 21st century competencies, promoting self-directed and collaborative learning through ICT-based lessons, and developing assessment to support learning.

It is clear at the start of the review that there is more to be considered than just focusing on the content. While there is a need to constantly review what students learn, the changes in content will not be the key lever. In fact, little has been changed in the content as this has stabilised over the years. Instead, more focus has now been given to skills and competencies that will make a better 21st century learner – the process of learning becomes more important than just what is to be taught and remembered. The syllabuses are therefore written with the view that not only will it inform teachers on what to teach, it will also influence the way teachers teach and students learn. One key feature of this set of syllabuses is the explication of learning experiences, besides the learning outcomes. This gives guidance to teachers on the opportunities that students should be given as part of their learning. Ultimately, how students learn matters.

Curriculum review and design is ongoing work. The quality of the curriculum is as much in its design as it is in its implementation. Teachers, who are the frontline of curriculum delivery, must believe in the value of the changes. Support, resources and training will be provided to build capacity in our teachers. All these will be part of the continuous effort to deliver the best mathematics curriculum for the students.

The O- & N(A)-Level Additional Mathematics syllabuses will be implemented starting from Secondary Three in 2013. The implementation schedule is as follows:

Year	2013	2014
Level	Sec 3	Sec 4

Goals and Aims

Different Syllabuses, Different Aims

The overarching goal of the mathematics curriculum is to ensure that all students will achieve a level of mastery of mathematics that will serve them well in life, and for those who have the interest and ability, to pursue mathematics at the highest possible level.

The broad aims of mathematics education in Singapore are to enable students to:

- acquire and apply mathematical concepts and skills;
- develop cognitive and metacognitive skills through a mathematical approach to problem solving; and
- develop positive attitudes towards mathematics.

The mathematics curriculum comprises a set of syllabuses spanning 12 years, from primary to pre-university, and is compulsory up to the end of secondary education. Each syllabus has its own specific set of aims to guide the design and implementation of the syllabus. The aims also influence the choice of content, skills as well as contexts to meet the specific needs of the students at the given level or course. Each syllabus expands on the three broad aims of mathematics education differently to cater for the different needs and abilities of the students (see table of aims on the next page).

What does it mean to teachers?

Understanding the aims of the syllabus helps teachers stay focused on the larger outcomes of learning and guides teachers when they embark on the school-based curriculum innovations and customisations.

Overview of Aims Across the Levels

Primary
Laying a Strong Foundation

The Primary Mathematics Syllabus aims to enable all students to:

- acquire mathematical concepts and skills for everyday use and for continuous learning in mathematics;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving;
- build confidence and foster interest in mathematics.

Secondary
Building Up Strengths

The O/N(A)-Level Mathematics Syllabus aims to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving;
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics;
- build confidence and foster interest in mathematics.

The O/N(A)-Level Additional Mathematics Syllabus aims to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, in particular, the sciences;
- develop thinking, reasoning and meta-cognitive skills through a mathematical approach to problem-solving;
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and
- appreciate the abstract nature and power of mathematics.

The N(T)-Level Mathematics Syllabus aims to enable students who are bound for post-secondary vocational education to:

- acquire mathematical concepts and skills for real life, to support learning in other subjects, and to prepare for vocational education;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence in using mathematics and appreciate its value in making informed decisions in real life.

Pre-University
Gearing Up for University Education

The H1 Mathematics Syllabus aims to enable students who are interested in pursuing tertiary studies in business and the social sciences to:

- acquire mathematical concepts and skills to support their tertiary studies in business and the social sciences;
- develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;
- connect ideas within mathematics and between mathematics and other disciplines through applications of mathematics;
- appreciate the value of mathematics in making informed decisions in life.

The H2 Mathematics Syllabus aims to enable students who are interested in pursuing tertiary studies in mathematics, sciences and engineering to:

- acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences and engineering;
- develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving and the use of mathematics language;
- connect ideas within mathematics and between mathematics and other disciplines through applications of mathematics;
- appreciate the beauty of mathematics and its value in making informed decisions in life.

The H3 Mathematics Syllabus aims to enable students who have an aptitude and passion for mathematics to:

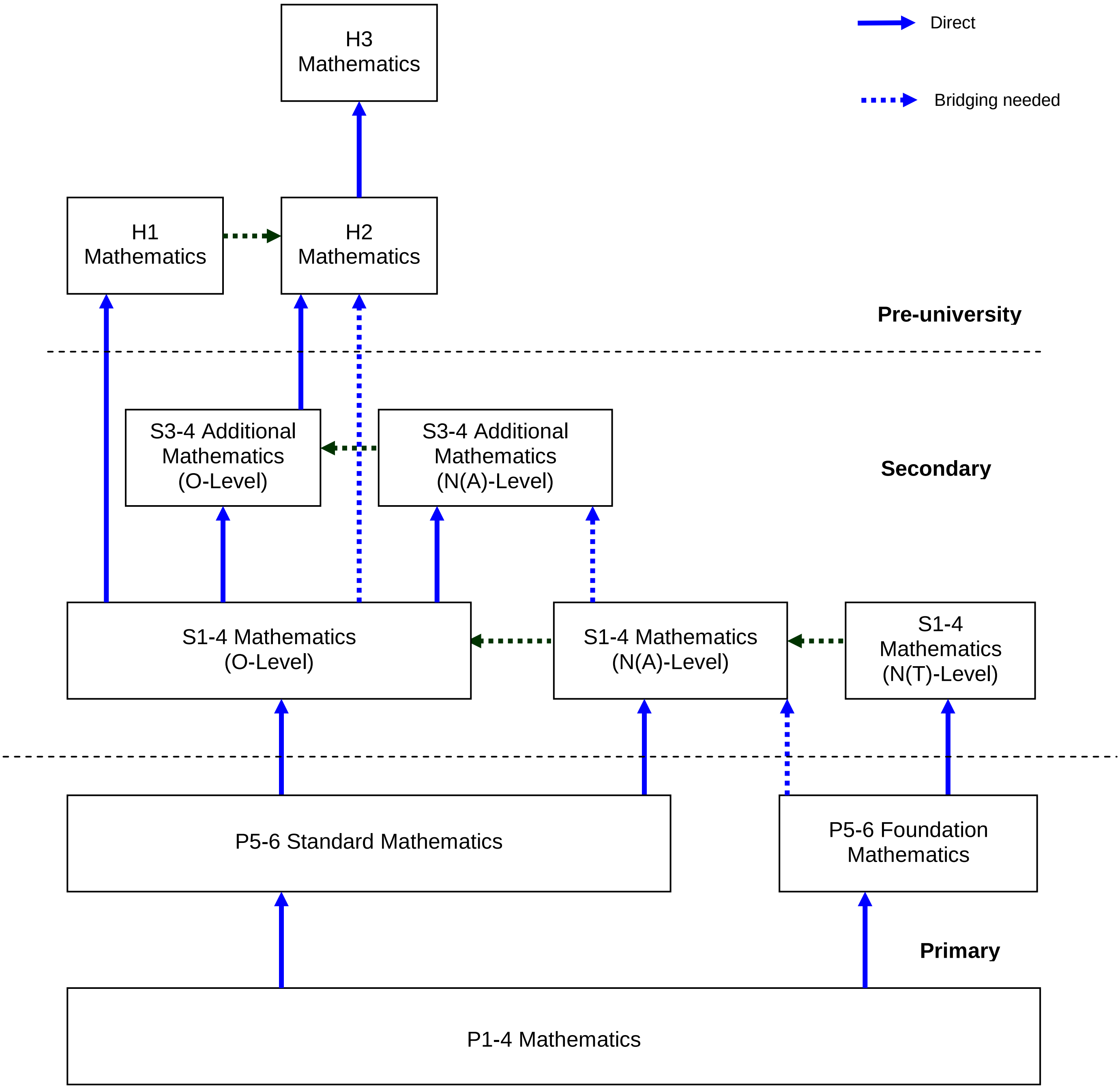
- acquire advanced mathematical concepts and skills to deepen their understanding of mathematics, and to widen the scope of applications of mathematics;
- develop rigorous habits of mind through mathematical reasoning and proof, creative mathematical problem solving, and use of mathematical models;
- connect ideas within mathematics at a higher level and between mathematics and other disciplines through applications of mathematics;
- appreciate the beauty, rigour and abstraction of mathematics through mathematical proof and applications.

Syllabus Design

Spiral Curriculum, Connected Syllabuses

Mathematics is largely hierarchical in nature. Higher concepts and skills are built upon the more foundational ones and have to be learned in sequence. A spiral approach is adopted in the building up of content across the levels.

The mathematics curriculum consists of a set of connected syllabuses to cater to the different needs and abilities of students. This section gives an overview of the syllabuses and their connections so that teachers are better able to appreciate the mathematics curriculum as a whole.



The Primary Mathematics syllabus assumes no formal learning of mathematics. However, basic pre-numeracy skills such as matching, sorting and comparing are necessary in providing a good grounding for students to begin learning at Primary 1 (P1).

The P1-4 syllabus is common to all students. The P5-6 Standard Mathematics syllabus continues the development of the P1-4 syllabus whereas the P5-6 Foundation Mathematics syllabus re-visits some of the important concepts and skills in the P1-4 syllabus. The new concepts and skills introduced in Foundation Mathematics is a subset of the Standard Mathematics syllabus.

The O-Level Mathematics syllabus builds on the Standard Mathematics syllabus. The N(A)-Level Mathematics syllabus is a subset of O-Level Mathematics, except that it re-visits some of the topics in Standard Mathematics syllabus. The N(T)-Level Mathematics syllabus builds on the Foundation Mathematics syllabus.

The O-Level Additional Mathematics syllabus assumes knowledge of O-Level Mathematics content and includes more in-depth treatment of important topics. The N(A)-Level Additional Mathematics syllabus is a subset of O-Level Additional Mathematics syllabus. O-Level Additional Mathematics together with O-Level Mathematics content provides the prerequisite knowledge required for H2 Mathematics at the pre-university level.

At the pre-university level, mathematics is optional. The H1 Mathematics syllabus builds on the O-level Mathematics syllabus. H2 Mathematics assumes some of the O-Level Additional Mathematics content. H3 Mathematics is an extension of H2 Mathematics.

Flexibility and Choice

There are two mathematics syllabuses at the P5-6 level. Most students would offer Standard Mathematics and for students who need more time to learn, they could offer Foundation Mathematics.

There are five mathematics syllabuses in the secondary mathematics curriculum. O-Level Mathematics, N(A)-Level Mathematics and N(T)-Level Mathematics provide students from the respective courses the core mathematics knowledge and skills in the context of a broad-based education. The more mathematically able students from the N(A) course can choose to take O-Level Mathematics in four years instead of five years. Likewise, the more able N(T) course students can also offer N(A)-Level Mathematics. The variation in pace and syllabus adds to the flexibility and choice within the secondary mathematics curriculum. At the upper secondary level, students who are interested in mathematics and are more mathematically inclined may choose to offer Additional Mathematics as an elective at the O-Level or N(A)-Level. This gives them the opportunity to learn more mathematics that would prepare them well for courses of study that require higher mathematics.

For students who wish to study in the Engineering-type courses at the polytechnics, Additional Mathematics will be a good grounding. The N(A)-Level and N(T)-Level Mathematics syllabuses will prepare students well for ITE courses. Students who aspire to study Mathematics or mathematics-related courses at the universities could offer H2 Mathematics, and if possible, H3 Mathematics.

What does it mean to teachers?

Teachers need to have the big picture in mind so that they can better understand the role of each syllabus, the connection it makes with the next level and the dependency relationship between syllabuses. This enables teachers to better understand what they have to do at their level, as well as to plan and advise students in their learning of mathematics. For example, H2 Mathematics assumes some of the O-Level Additional Mathematics content but may be offered by students without Additional Mathematics background as long as effort is made to bridge the gap.

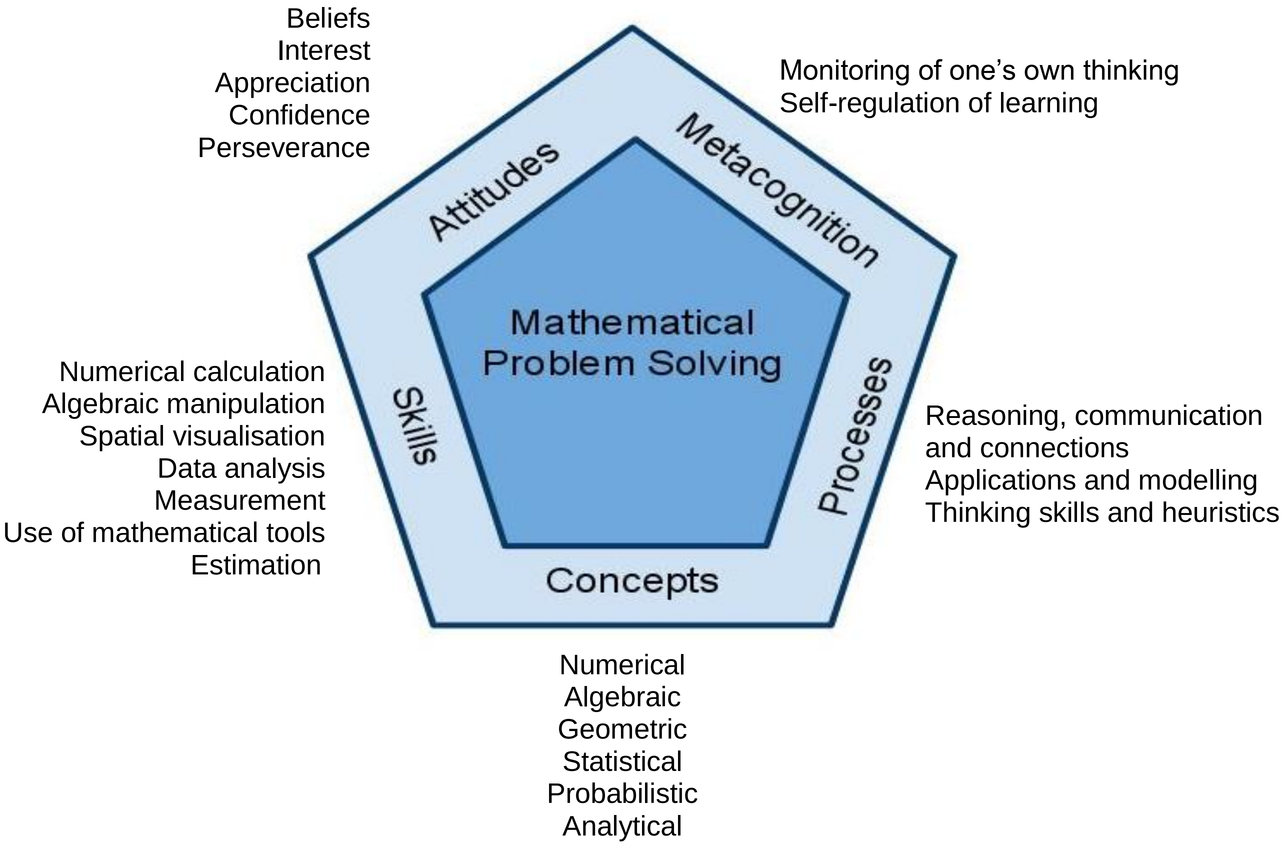
Chapter 2
Mathematics Framework

Problem Solving

Problem Solving

Concepts, Skills, Processes, Metacognition, Attitudes

The Mathematics Framework has been a feature of our mathematics curriculum since 1990, and is still relevant to date. The central focus of the framework is mathematical problem solving, that is, using mathematics to solve problems. The framework sets the direction for and provides guidance in the teaching, learning, and assessment of mathematics at all levels, from primary to pre-university. It reflects also the 21st century competencies[1].



The framework stresses *conceptual understanding*, *skills proficiency* and *mathematical processes*, and gives due emphasis to *attitudes* and *metacognition*. These five components are inter-related.

Concepts

Mathematical concepts can be broadly grouped into *numerical*, *algebraic*, *geometric*, *statistical*, *probabilistic*, and *analytical* concepts. These content categories are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

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[1] Information on the MOE framework for 21st century competencies and student outcomes can be found on www.moe.gov.sg

To develop a deep understanding of mathematical concepts, and to make sense of various mathematical ideas as well as their connections and applications, students should be exposed to a variety of learning experiences including hands-on activities and use of technological aids to help them relate abstract mathematical concepts with concrete experiences.

Skills

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

To develop proficiencies in mathematics skills, students should have opportunities to use and practise the skills. These skills should be taught with an understanding of the underlying mathematical principles and not merely as procedures.

Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics and beyond.

In the context of mathematics, *reasoning, communication and connections* take on special meanings:

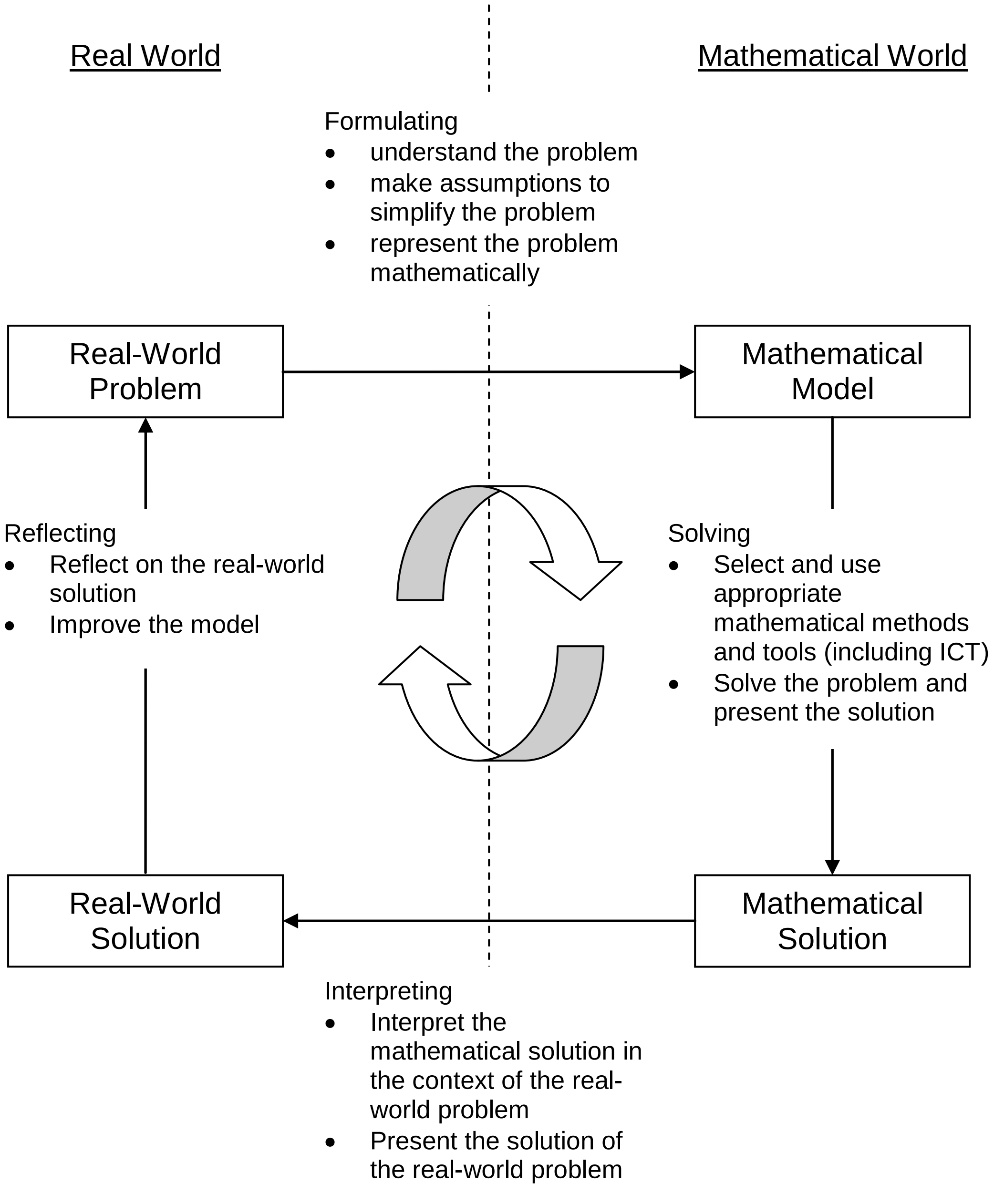
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments. It is a habit of mind that can be developed through application of mathematics in different contexts.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their understanding of mathematics and sharpen their mathematical thinking.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world. This helps students make sense of what they learn in mathematics.

Applications and modelling allow students to connect mathematics that they have learnt to the real world, enhance understanding of key mathematical concepts and methods as well as develop mathematical competencies. Students should have opportunities to apply mathematical problem-solving and reasoning skills to tackle a variety of problems, including open-ended and real-world problems. Mathematical modelling is the process of formulating and improving a mathematical model[2] to

[2] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

represent and solve real-world problems. Through mathematical modelling, students learn to deal with ambiguity, make connections, select and apply appropriate mathematical concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data.

Mathematical Modelling Process (version 2010)



Thinking skills and heuristics are essential for mathematical problem solving. Thinking skills are skills that can be used in a thinking process, such as classifying, comparing, analysing parts and whole, identifying patterns and relationships, induction, deduction, generalising, and spatial visualisation. Heuristics are general rules of thumb of what students can do to tackle a problem when the solution to the problem is not obvious. These include using a representation (e.g., drawing a diagram, tabulating), making a guess (e.g., trial and error/guess and check, making a supposition), walking through the process (e.g., acting it out, working backwards) and changing the problem (e.g., simplifying the problem, considering special cases).

Metacognition

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

To develop metacognitive awareness and strategies, and know when and how to use the strategies, students should have opportunities to solve non-routine and open-ended problems, to discuss their solutions, to think aloud and reflect on what they are doing, and to keep track of how things are going and make changes when necessary.

Attitudes

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;
- confidence in using mathematics; and
- perseverance in solving a problem.

Students' attitudes towards mathematics are shaped by their learning experiences. Making the learning of mathematics fun, meaningful and relevant goes a long way to inculcating positive attitudes towards the subject. Care and attention should be given to the design of the learning activities to build confidence in and develop appreciation for the subject. Above all, students' beliefs can influence their attitudes in learning, especially in student-centred learning where students are encouraged to take on more responsibility for their own learning.

What does it mean to teachers?

The five components of the Mathematics Framework are integral parts of mathematics learning and problem solving. The intent of the framework is to help teachers focus on these components in their teaching practice so as to provide a more engaging, student-centred, and technology-enabled learning environment, and to promote greater diversity and creativity in learning.

Chapter 3
Teaching, Learning And Assessment

Learning Experiences
Teaching and Learning
Assessment in the Classroom

Learning Experiences

It matters how students learn

Learning mathematics is more than just learning concepts and skills. Equally important are the cognitive and metacognitive process skills. These processes are learned through carefully constructed learning experiences. For example, to encourage students to be inquisitive, the learning experiences must include opportunities where students discover mathematical results on their own. To support the development of collaborative and communication skills, students must be given opportunities to work together on a problem and present their ideas using appropriate mathematical language and methods. To develop habits of self-directed learning, students must be given opportunities to set learning goals and work towards them purposefully. A classroom, rich with these opportunities, will provide the platform for students to develop these 21st century competencies.

Learning experiences are stated in the mathematics syllabuses to influence the ways teachers teach and students learn so that the curriculum objectives can be achieved. These statements expressed in the form "students should have opportunities to …" remind teachers of the student-centric nature of these experiences. They describe actions that students will perform and activities that students will go through, with the opportunities created and guidance rendered by teachers. The descriptions are sufficiently specific to provide guidance yet broad enough to give flexibility to the teachers.

For each topic, the learning experiences focus on the mathematical processes and skills that are integral parts of learning of that topic. There are also generic learning experiences that focus on the development of good learning habits and skills such as:

Students should have opportunities to:

- take notes and organise information meaningfully;
- practise basic mathematical skills to achieve mastery;
- use feedback from assessment to improve learning;
- solve novel problems using a repertoire of heuristics;
- discuss, articulate and explain ideas to develop reasoning skills; and
- carry out a modelling project.

These learning experiences, whether they are topical or generic, are not exhaustive. Teachers are encouraged to do more to make learning meaningful and effective.

Teaching and Learning

Principles of Teaching and Phases of Learning

This section outlines three principles of mathematics teaching and the three phases of mathematics learning in the classrooms.

Principles of Teaching

Principle 1
Teaching is for learning; learning is for understanding; understanding is for reasoning and applying and, ultimately problem solving.

Teaching is an interactive process that is focused on students' learning. In this process, teachers use a range of teaching approaches to engage students in learning; students provide teachers with feedback on what they have learnt through assessment; and teachers in turn provide feedback to students and make decisions about instructions to improve learning.

The learning of mathematics should focus on understanding, not just recall of facts or reproduction of procedures. Understanding is necessary for deep learning and mastery. Only with understanding can students be able to reason mathematically and apply mathematics to solve a range of problems. After all, problem solving is the focus of the mathematics curriculum.

Principle 2
Teaching should build on students' knowledge; take cognisance of students' interests and experiences; and engage them in active and reflective learning.

Mathematics is a hierarchical subject. Without understanding of pre-requisite knowledge, foundation will be weak and learning will be shallow. It is important for teachers to check on students' understanding before introducing new concepts and skills.

Teachers need to be aware of their students' interests and abilities so as to develop learning tasks that are stimulating and challenging. This is important in order to engage students in active and reflective learning where students participate and take ownership of the learning.

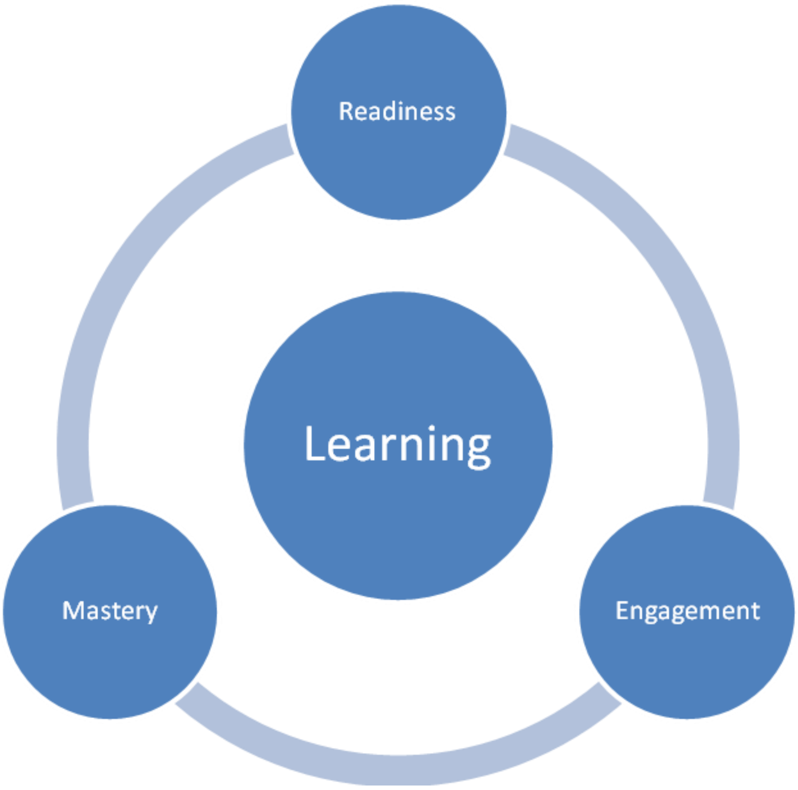
Principle 3
Teaching should connect learning to the real world, harness ICT tools and emphasise 21st century competencies.

There are many applications of mathematics in the real world. Students should have an understanding and appreciation of these applications and how mathematics is used to model and solve problems in real-world contexts. In this way, students will see the meaning and relevance of mathematics.

Teachers should consider the affordances of ICT to help students learn. ICT tools can help students understand mathematical concepts through visualisations, simulations and representations. They can also support exploration and experimentation and extend the range of problems accessible to students. The ability to use ICT tools is part of the 21st century competencies. It is also important to design learning in ways that promote the development of other 21st century competencies such as working collaboratively and thinking critically about the mathematical solution.

Phases of Learning

Effective instruction of a unit typically involves three phases of learning: *Readiness*, *Engagement* and *Mastery*.



Phase 1 - *Readiness*

Student readiness to learn is vital to learning success. In the readiness phase of learning, teachers prepare students so that they are ready to learn. This requires considerations of *prior knowledge*, *motivating contexts*, and *learning environment*.

- ***Prior Knowledge***
  For students to be ready to learn, teachers need to know students' prior knowledge in relation to the new learning. This requires knowing whether students have the pre-requisite concepts and skills. Some form of diagnostic assessment is necessary to check that students are ready to learn.

- ***Motivating Contexts***
  For students to be ready to learn, teachers need to provide motivating contexts for learning. These contexts should be developmentally appropriate. For example, younger students may like contexts such as stories and songs, and play-based activities such as games, whereas older students may appreciate contexts related to everyday life so that they can see the relevance and meaningfulness of mathematics. For the more advanced students, applications in other disciplines can serve as motivation for learning.

- ***Learning Environment***
  Shared rules help promote respectful and emotionally-safe interactions between teacher and students and among students that are necessary for productive and purposeful learning. Established procedures for organising students and managing resources will also facilitate a smooth start and transitions during lessons.

Phase 2 - *Engagement*

This is the main phase of learning where teachers use a repertoire of pedagogies to engage students in learning new concepts and skills. Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. For example, the lesson or unit could start with an activity, followed by teacher-led inquiry and end with direct instruction.

- ***Activity-based Learning***
  This approach is about learning by doing. It is particularly effective for teaching mathematical concepts and skills at primary and lower secondary levels, but is also effective at higher levels. Students engage in activities to explore and learn mathematical concepts and skills, individually or in groups. They could use manipulatives or other resources to construct meanings and understandings. From concrete manipulatives and experiences, students are guided to uncover abstract mathematical concepts or results.

  For example, this approach can be used to draw connections between the different representations of *Exponential and Logarithmic Functions.* Students are divided into groups and provided with three sets of cards: one set which contains the graphs of the functions, the second set contains the equations and the third set contains the data. The cards may also include graphs from real world or scientific contexts. Through the matching activity, students discuss the main characteristics of these functions in graphical, tabular and algebraic forms. During the course of the activity the teacher provides support and intervention if necessary. The teacher concludes by providing opportunities for students to justify their responses and consolidate their learning.

- ***Teacher-directed Inquiry***
  This approach is about learning through guided inquiry. Instead of giving the answers, teachers lead and involve students in exploration and investigation. Students learn to focus on specific questions and ideas and are engaged in communicating, explaining and reflecting on their answers. They also learn to pose questions, process information and data and seek appropriate methods and solutions. This enhances the development of mathematical processes and 21^{st} century competencies.

  For example, this approach can be used to introduce *Linear Law*. Teacher plots a graph of a non-linear equation using a given set of data. Teacher then asks the students to discuss and explore how this equation can be transformed such that its graph is linear. Teacher guides the students through the process by providing scaffolding questions that lead to the manipulation of the non-linear equation to a linear form $Y = mX + c$.

- ***Direct Instruction***
  This approach is about explicit teaching. Teachers introduce, explain and demonstrate new concepts and skills. Direct instruction is most effective when students are told what they will be learning and what they are expected to be able to do. This helps them focus on the learning goals. Teachers draw connections, pose questions, emphasise key concepts, and role-model thinking. Holding students' attention is critical. Stimuli such as videos, graphic images, real-world contexts, and even humour, aid in maintaining a high level of attention.

  For example, this approach can be used to introduce the concept of derivative when students learn *Differentiation*. The teacher starts by explaining the geometrical meaning and the physical meaning of derivative. The teacher transfers the knowledge by making references to the gradient of lines and helps students to understand the meaning of gradient of a curve at a given point using different graphs. To deepen conceptual understanding, the teacher asks students to distinguish between constant rate of change, average rate of change and instantaneous rate of change. The teacher assesses students' understanding, identifies gaps and misconceptions and emphasises key concepts. The teacher draws examples from sciences and the real world to illustrate and help students appreciate the applications of the derivatives.

Phase 3 - *Mastery*

This is the final phase of learning where teachers help students consolidate and extend their learning. The mastery approaches include:

- ***Motivated Practice***
  Students need practice to achieve mastery. Practice can be motivating and fun. Practice must include repetition and variation to achieve proficiency and flexibility. Structuring practice in the form of games is one

good strategy to make practice motivating and fun, while allowing for repetition and variation. There should be a range of activities, from simple recall of facts to application of concepts.

- ***Reflective Review***
  It is important that students consolidate and deepen their learning through tasks that allow them to reflect on their learning. This is a good habit that needs to be cultivated from an early age and it supports the development of metacognition. Summarising their learning using concept maps, writing journals to reflect on their learning and making connections between mathematical ideas and between mathematics and other subjects should be encouraged. Sharing such reflections through blogs makes learning social.

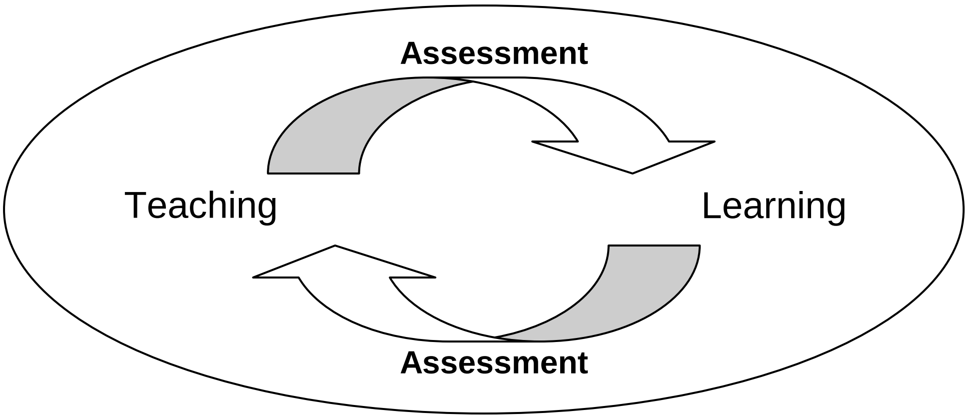
- ***Extended Learning***
  Students who are mathematically inclined should have opportunities to extend their learning. These can be in the form of more challenging tasks that stretch their thinking and deepen their understanding.

Assessment in the Classroom

Supporting Teaching and Learning in Mathematics

Role of assessment

Assessment is an integral part of the interactive process of teaching and learning, as illustrated in the diagram below. It is an ongoing process by which teachers gather information about students' learning to inform and support teaching. An important product of assessment is feedback. Feedback must be timely and rich. It must inform students where they are in their learning and what they need to do to improve their learning. It must also inform teachers what they need to do to address learning gaps and how to improve their instruction.



Range of assessment

Assessments can be broadly classified as summative, formative, and diagnostic.

- Summative assessments, such as tests and examinations, measure what students have learned. Teachers usually report the assessment result as a score or a grade.

- Formative and diagnostic assessments are used as assessment for learning to provide timely feedback to students on their learning, and to teachers on their teaching.

Assessment in the classroom should focus on helping students improve their learning. Therefore, they are primarily formative and diagnostic in purpose.

Though teachers are comfortable with the use of traditional pen-and-paper tests to find out how much students know and can do, there is value in exploring a wider variety of assessment strategies. These strategies allow teachers to gather information that is not easily available through traditional methods of assessment, but are nevertheless valuable in supporting learning. Ultimately, the choice of assessment strategies must be guided by its purpose, that is, it must be fit-for-purpose.

Integrating assessment with instruction

It is important that teachers know what and when to assess student learning, and how to embed the assessment in the learning process. Assessment can be integrated into classroom discourse and activities using different assessment strategies. For example, teachers may watch students solve problems and get them to explain their strategies. Teachers may also engage students in assessing their own work and reflecting on their own learning and how to improve it. Both moment-by-moment assessment and planned assessment should be considered.

Effective questioning can scaffold learning and probe understanding. It creates teachable moments for teachers to correct a misconception, reinforce a point or expand on an idea. The questions can be open-ended to encourage students to consider alternative approaches. Sufficient wait-time is necessary so that students can formulate their thoughts, communicate and share their ideas, and hear the ideas of others. In the process, students learn to articulate their thinking and deepen their understanding, and develop confidence in talking about mathematics and using it. Teachers can assess students' thinking and understanding, and provide useful feedback to improve their learning.

Teachers can integrate performance assessments into the instructional process to provide additional learning experiences for students. This type of assessment requires students to apply their knowledge and skills in context, and the focus is on mathematical processes rather than on mathematics content. A rubric is useful to show teachers what to look for in students' work, but more importantly, it shows what is expected of students in terms of processes and quality of work. The rubric also provides a structured means of giving qualitative feedback. Teachers may allow students to assess their own performances so that they can reflect on their work and make improvements.

Assessment for learning calls for new ways of assessment in the classroom. It involves a change in teachers' roles and in the expectations of students. By integrating assessment and instruction, students will be more engaged in and will take greater ownership of their learning.

Chapter 4
O-Level Additional Mathematics Syllabus

Aims of Syllabus
Syllabus Organisation
Mathematical Processes
Content and Learning Experiences

Aims of Syllabus

The O-Level Additional Mathematics syllabus aims to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, in particular, the sciences;
- develop thinking, reasoning and metacognitive skills through a mathematical approach to problem-solving;
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and
- appreciate the abstract nature and power of mathematics.

Syllabus Organisation

The syllabus is organised along 3 content strands with a listing of mathematical processes that cut across the 3 strands.

<table>
<tr><td colspan="3">3 Content + 1 Process Strand</td></tr>
<tr><td>Algebra</td><td>Geometry and Trigonometry</td><td>Calculus</td></tr>
<tr><td colspan="3">Mathematical Processes</td></tr>
</table>

Strand: Mathematical Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes reasoning, communication and connections, applications and modelling, and thinking skills and heuristics that are important in mathematics and beyond.

In Additional Mathematics, students should be given opportunities to reason with and about Mathematics and connect new learning to prior knowledge. For example, in Algebra, students should be given opportunities to make connection between the different representations of functions, analyse the effects of varying coefficients and parameters in a given representation of function and use them to justify their thinking and present their finding. In Geometry, students will move from inductive reasoning in O-Level Mathematics to deductive reasoning in Additional Mathematics. Students should have opportunities to understand the power of deductive proof in establishing results and are expected to write formal proofs to explain their reasoning and to solve geometry problems.

While the focus in O-Level Mathematics is on developing an awareness of the mathematical modelling process, in Additional Mathematics, students should be given opportunities to work on problems that require them to have an understanding of mathematical models, represent solutions in an appropriate mathematical form, and interpret solution with respect to the given context. These problems could be drawn from disciplines such as physical and life sciences. For example, students should be able to appreciate that the phenomena with periodic feature could be modelled by trigonometric functions and population growth by exponential functions.

The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills. Students should be exposed to problem solving approach such as the Polya's model. Teachers could "think aloud" to give attention to these processes and make them visible to students. Students should be given opportunity to work in groups and use ICT tools for modelling tasks. ICT tools empower students to work on problems which would otherwise require more advanced mathematics or computations that are too tedious and repetitive. The learning environment should provide opportunities for students to work collaboratively on mathematical tasks, present and justify their findings and engage in peer critique. It should also encourage communication; both oral and written.

No.	Processes	Details
MATHEMATICAL PROCESSES		
MP1	**Reasoning, Communication and Connections**	• Reason inductively and deductively, including: ∗ Explaining or justifying, or critiquing a mathematical solution/statement ∗ Drawing logical conclusions ∗ Making inferences ∗ Writing mathematical arguments and proofs • Use appropriate representations, mathematical language (including notations, symbols and conventions) and technology to present and communicate mathematical ideas • Make connections within mathematics, between mathematics and other disciplines, and between mathematics and the real world
MP2	**Application and Modelling**	• Apply mathematics concepts and skills to solve problems in a variety of contexts within or outside mathematics, including: ∗ Identify the appropriate mathematical representations or standard models for a problem ∗ Use appropriate mathematical concepts, skills (including tools and algorithm) to solve a problem • Stages of modelling ∗ Understanding a real-world problem ∗ Formulating a real-world problem into a mathematical one by making suitable assumption and simplification and identifying suitable mathematical representations ∗ Applying mathematics to solve the real-world problem ∗ Interpreting the mathematical solution in the context of the real- world problem, including verifying against real data ∗ Refining and improving the model
MP3	**Thinking Skills and Heuristics**	• Use thinking skills such as: ∗ Classifying ∗ Comparing ∗ Sequencing ∗ Generalising ∗ Induction ∗ Deduction ∗ Analyzing (from whole to parts) ∗ Synthesizing (from parts to whole) • Use a problem-solving model such as Polya's model • Use heuristics such as: ∗ Drawing a diagram ∗ Tabulating ∗ Trial and error ∗ Guess and check ∗ Acting it out ∗ Working backwards ∗ Simplifying a problem ∗ Considering special cases

Content and Learning Experiences

The content is divided into three strands viz. Algebra, Geometry and Trigonometry, and Calculus. In addition to the details for each content strand, the syllabus also includes learning experiences.

The learning experiences for O-Level Additional Mathematics should provide opportunities for students to

- enhance conceptual understanding through use of various mathematical tools including ICT tools;
- make connections between representations, topics and methods;
- reason and communicate using mathematics language;
- apply mathematics knowledge and skills to real-world and other disciplines; and
- appreciate the beauty and value of mathematics.

Content	Learning Experiences
ALGEBRA	**Students should have opportunities to:**
A1 Equations and inequalities	
1.1 Conditions for a quadratic equation to have: ∗ two real roots ∗ two equal roots ∗ no real roots and related conditions for a given line to: ∗ intersect a given curve ∗ be a tangent to a given curve ∗ not intersect a given curve 1.2 Conditions for ax^2+bx+c to be always positive (or always negative) 1.3 Solving simultaneous equations in two variables with at least one linear equation, by substitution 1.4 Relationships between the roots and coefficients of a quadratic equation 1.5 Solving quadratic inequalities, and representing the solution on the number line	(a) Explain how the roots of the equation $ax^2+bx+c=0$ are related to the sign of b^2-4ac. (b) Transform ax^2+bx+c to the form $a(x-h)^2+k$ and use it to (i) sketch the graph; and (ii) deduce the quadratic formula. (c) Use a graphing software to investigate how the positions of the graph $y=ax^2+bx+c$ vary with the sign of b^2-4ac, and describe the graph when b^2-4ac < 0. (d) Use a graphing software to investigate the relationship between the number of points of intersection and the nature of solutions of a pair of simultaneous equations, one linear and one quadratic. (e) Examine the solution of a quadratic equation and that of its related quadratic inequality (e.g. $4x^2+x-5=0$ and $4x^2+x-5>0$), and describe both solutions and their relationship.

Content	Learning Experiences
ALGEBRA	**Students should have opportunities to:**
A2 Indices and surds	
2.1 Four operations on indices and surds, including rationalising the denominator 2.2 Solving equations involving indices and surds	(a) Make sense of numbers in surd form and recognise that the quadratic formula gives the real roots of quadratic equations in various forms (integer, rational number and conjugate surds).
A3 Polynomials and Partial Fractions	
3.1 Multiplication and division of polynomials 3.2 Use of remainder and factor theorems 3.3 Factorisation of polynomials 3.4 Use of: $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$ 3.5 Solving cubic equations 3.6 Partial fractions with cases where the denominator is no more complicated than: • $(ax+b)(cx+d)$ • $(ax+b)(cx+d)^2$	(a) Make connections between division of polynomial and division of whole number, and express the division algorithm as P(x) = (x – a)Q(x) + R. (b) Associate the remainder and factor theorems with the division algorithm. (c) Use a graphing software to investigate the graph of a cubic polynomial and discuss (i) the linear factors of the polynomial and the number of real roots; and (ii) the number of real roots of the related cubic equation, with reference to the points of intersection with the x-axis. (d) Combine two or three proper algebraic fractions into a single fraction, and vice versa, i.e. to express the single fraction as partial fractions. (e) Differentiate between proper and improper fractions, and express an improper fraction as a polynomial and a proper fraction.

Content	Learning Experiences
ALGEBRA	**Students should have opportunities to:**
• $(ax+b)(x^2+c^2)$	
A4 Binomial expansions	
4.1 Use of the Binomial Theorem for positive integer n 4.2 Use of the notations $n!$ and $\binom{n}{r}$ 4.3 Use of the general term $\binom{n}{r}a^{n-r}b^r$, $0 < r \leq n$ (knowledge of the greatest term and properties of the coefficients is not required)	(a) Expand $(a+b)^n$ for $n = 2,3,4,....$ and generalise the result to the binomial theorem. (b) Make connections between the coefficients of the binomial expansion and the Pascal Triangle, and use it to find the coefficients when n is small.
A5 Power, Exponential, Logarithmic and Modulus Functions .	
5.1 Power functions $y = ax^n$, where n is a simple rational number and their graphs 5.2 Exponential and logarithmic functions a^x, e^x, $log_a x$, *In x* and their graphs, including ▪ laws of logarithms	(a) Use a graphing software to explore the characteristics of various functions. (b) Relate the solution of the equation f(x) = 0 to the graph y = f(x) to verify the existence of the solutions or to justify that the solution does not exist. (c) Use a graphing software to display real-world data graphically and match it with an appropriate function. (d) Relate the exponential and logarithmic functions to sciences (e.g. pH value, Richter

Content	Learning Experiences
ALGEBRA	**Students should have opportunities to:**
▪ equivalence of $y = a^x$ and $x = \log_a y$ ▪ change of base of logarithms 5.3 Modulus functions $\lvert x \rvert$ and $\lvert \mathrm{f}(x) \rvert$, where $\mathrm{f}(x)$ is linear, quadratic or trigonometric, and their graphs 5.4 Solving simple equations involving exponential, logarithmic and modulus functions.	scale of earthquakes, decibel scale for sound intensity, radioactive decay, population growth).

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
G1 Trigonometric functions, identities and equations	
1.1 Six trigonometric functions for angles of any magnitude (in degrees or radians) 1.2 Principal values of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ 1.3 Exact values of the trigonometric functions for special angles $(30°, 45°, 60°)$ or $(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3})$ 1.4 Amplitude, periodicity and symmetries related to sine and cosine functions 1.5 Graphs of $y = a\sin(bx) + c$, $y = a\sin(\frac{x}{b}) + c$, $y = a\cos(bx) + c$, $y = a\cos(\frac{x}{b}) + c$ and $y = a\tan(bx)$, where *a* is real, *b* is a positive integer and *c* is an integer. 1.6 Use of: * $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$,	(a) Discuss the relationships between $\sin A, \cos A$ and $\tan A$, with respect to the line segments related to a unit circle. (b) Use a graphing software to display the graphs of trigonometric functions and discuss their behaviours, and investigate how a graph (e.g. $y = a\sin bx + c$) changes when a, b or c varies. (c) Relate the sine and cosine functions to sciences (e.g. tides, Ferris wheel and sound waves). (d) Relate $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$ to the sine, cosine and tangent functions respectively (e.g. $\sin^{-1}x$ is an angle whose sine is x, and the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is -30° or $-\frac{\pi}{6}$). (e) Deduce the formula for sin (A+B) from the sine rule.

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
$\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$ ∗ the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ ∗ the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ ∗ the expression of $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$ 1.7 Simplification of trigonometric expressions 1.8 Solution of simple trigonometric equations in a given interval (excluding general solution) 1.9 Proofs of simple trigonometric identities	
G2 Coordinate geometry in two dimensions	
2.1 Condition for two lines to be parallel or perpendicular 2.2 Midpoint of line segment 2.3 Area of rectilinear figure	(a) Relate the gradient of a straight line to the tangent of the angle between the line and the positive direction of the x-axis, and deduce the relationship between the gradients of (i) two parallel lines and (ii) two perpendicular lines. (b) Discuss how to solve geometry problems involving finding (i) the equation of a line perpendicular or parallel to a given line, (ii) the coordinates of the midpoint of a line segment (horizontal, vertical and oblique), and (iii) equation of the perpendicular bisector of a line segment.

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
	(c) Explore and discuss ways of finding the area of a triangle (or polygon) with given vertices.
2.4 Graphs of parabolas with equations in the form $y^2 = kx$ 2.5 Coordinate geometry of circles in the form: • $(x-a)^2 + (y-b)^2 = r^2$ • $x^2 + y^2 + 2gx + 2fy + c = 0$ (excluding problems involving two circles)	(d) Use a graphing software to investigate the graph of $y^2 = kx$ when k varies. (e) Relate parabolas to examples in sciences and in the real world. (f) Make connections between the graphs of $y^2 = x$ and $y = x^2$. (g) Derive the equation of a circle with centre (a, b) and radius r using the Pythagoras theorem, and the special case when the centre is at the origin. (h) Discuss how to solve geometry problems involving intersection of a parabola/circle and a straight line.
2.6 Transformation of given relationships, including $y = ax^n$ and $y = kb^x$, to linear form to determine the unknown constants from a straight line graph.	(i) Explain the use of a straight line graph in a science experiment (e.g. oscillation of a pendulum, Hooke's Law, Ohm's Law).

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
G3 Proofs in plane geometry	
Use of: 3.1 properties of parallel lines cut by a transversal, perpendicular and angle bisectors, triangles, special quadrilaterals and circles♦ 3.2 congruent and similar triangles♦ 3.3 midpoint theorem 3.4 tangent-chord theorem (alternate segment theorem)	(a) Discuss properties of triangles, quadrilaterals and circles, and justify and explain whether a geometrical statement is true. (b) Deduce basic geometric properties such as 'the base angles of an isosceles triangle are equal' from congruent triangles. (c) Explain the logical steps in a proof using appropriate language, definitions and theorems.

♦ These are properties learnt in O-Level Mathematics.

Content	Learning Experiences
CALCULUS	**Students should have opportunities to:**
C1 Differentiation and integration	
1.1 Derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y=\mathrm{f}(x)$ at a point 1.2 Derivative as rate of change 1.3 Use of standard notations $\mathrm{f}'(x), \mathrm{f}''(x), \dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}[=\dfrac{d}{dx}(\dfrac{dy}{dx})]$ 1.4 Derivatives of x^n, for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x, and $\ln x$, together with constant multiples, sums and differences 1.5 Derivatives of products and quotients of functions 1.6 Derivatives of composite functions 1.7 Increasing and decreasing functions 1.8 Stationary points (maximum and minimum turning points and	(a) Relate the derivative of a function to the gradient of the tangent to a curve at a given point, including horizontal and vertical tangents. (b) Distinguish between constant, average and instantaneous rate of change with reference to graphs. (c) Relate the sign of the first derivative of a function to the behaviour of the function (increasing or decreasing), locate points on the graph where the derivative is zero, and describe the behaviour of the function before, at and after these points. (d) Discuss cases where the second derivative test to discriminate between maxima and minima fails (e.g. $y=x^3$, $y=x^4$) and instead, use the first derivative test. (e) Discuss examples of problems in real-world contexts (e.g. business and sciences), involving the use of differentiation. (f) Explain what $\dfrac{d}{dx}(f(x))$, $\int f(x)dx$ and $\int_a^b f(x)dx$ represent and make connections between • derivative and indefinite integral; • definite and indefinite integrals. (g) Relate the area bounded by a curve and the y-axis to the area under the curve. (h) Model the motion of a particle in a straight line, using displacement, velocity and acceleration as vectors (e.g. velocity in the positive direction of x-axis is positive),

Content		Learning Experiences
CALCULUS		**Students should have opportunities to:**
	stationary points of inflexion)	and explain the physical meanings of $\frac{ds}{dt}$ and $\frac{dv}{dt}$, and their signs in relation to the motion.
1.9	Use of second derivative test to discriminate between maxima and minima	
1.10	Apply differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems	
1.11	Integration as the reverse of differentiation	
1.12	Integration of x^n for any rational n, $\sin x$, $\cos x$, $\sec^2 x$ and e^x, together with constant multiples, sums and differences	
1.13	Integration of $(ax+b)^n$ for any rational n, $\sin(ax+b)$, $\cos(ax+b)$ and $e^{(ax+b)}$	
1.14	Definite integral as area under a curve	
1.15	Evaluation of definite integrals	

Content	Learning Experiences
CALCULUS	**Students should have opportunities to:**
1.16 Finding the area of a region bounded by a curve and line(s) (excluding area of region between 2 curves) 1.17 Finding areas of regions below the x-axis 1.18 Application of differentiation and integration to problems involving displacement, velocity and acceleration of a particle moving in a straight line	

Chapter 5
N(A)-Level Additional Mathematics Syllabus

Aims of Syllabus
Syllabus Organisation
Mathematical Processes
Content and Learning Experiences

Aims of Syllabus

The N(A)-Level Additional Mathematics syllabus aims to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, in particular, the sciences;
- develop thinking, reasoning and metacognitive skills through a mathematical approach to problem-solving;
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and
- appreciate the abstract nature and power of mathematics.

Syllabus Organisation

The syllabus is organised along 3 content strands with a listing of mathematical processes that cut across the 3 strands.

3 Content + 1 Process Strand		
Algebra	Geometry and Trigonometry	Calculus
Mathematical Processes		

Strand: Mathematical Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes reasoning, communication and connections, applications and modelling, and thinking skills and heuristics that are important in mathematics and beyond.

In Additional Mathematics, students should be given opportunities to reason with and about Mathematics and connect new learning to prior knowledge. For example, in Algebra, students should be given opportunities to make connection between the different representations of functions, analyse the effects of varying coefficients and parameters in a given representation of function and use them to justify their thinking and present their finding.

While the focus in N(A)-Level Mathematics is on developing an awareness of the mathematical modelling process, in Additional Mathematics, students should be given opportunities to work on problems that require them to have an understanding of mathematical models, represent solutions in an appropriate mathematical form, and interpret solution with respect to the given context. These problems could be drawn from disciplines such as physical and life sciences. For example, students should be able to appreciate that the phenomena with periodic feature could be modelled by trigonometric functions and population growth by exponential functions.

The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills. Students should be exposed to problem solving approach such as the Polya’s model. Teachers could “think aloud” to give attention to these processes and make them visible to students. Students should be given opportunity to work in groups and use ICT tools for modelling tasks. ICT tools empower students to work on problems which would otherwise require more advanced mathematics or computations that are too tedious and repetitive. The learning environment should provide opportunities for students to work collaboratively on mathematical tasks, present and justify their findings and engage in peer critique. It should also encourage communication; both oral and written.

No.	Processes	Details
MATHEMATICAL PROCESSES		
MP1	**Reasoning, Communication and Connections**	• Reason inductively and deductively, including: ∗ Explaining or justifying, or critiquing a mathematical solution/statement ∗ Drawing logical conclusions ∗ Making inferences ∗ Writing mathematical arguments and proofs • Use appropriate representations, mathematical language (including notations, symbols and conventions) and technology to present and communicate mathematical ideas • Make connections within mathematics, between mathematics and other disciplines, and between mathematics and the real world
MP2	**Application and Modelling**	• Apply mathematics concepts and skills to solve problems in a variety of contexts within or outside mathematics, including: ∗ Identify the appropriate mathematical representations or standard models for a problem ∗ Use appropriate mathematical concepts, skills (including tools and algorithm) to solve a problem • Stages of modelling ∗ Understanding a real-world problem ∗ Formulating a real-world problem into a mathematical one by making suitable assumption and simplification and identifying suitable mathematical representations ∗ Applying mathematics to solve the real-world problem ∗ Interpreting the mathematical solution in the context of the real- world problem, including verifying against real data ∗ Refining and improving the model
MP3	**Thinking Skills and Heuristics**	• Use thinking skills such as: ∗ Classifying ∗ Comparing ∗ Sequencing ∗ Generalising ∗ Induction ∗ Deduction ∗ Analyzing (from whole to parts) ∗ Synthesizing (from parts to whole) • Use a problem-solving model such as Polya’s model • Use heuristics such as: ∗ Drawing a diagram ∗ Tabulating ∗ Trial and error ∗ Guess and check ∗ Acting it out ∗ Working backwards ∗ Simplifying a problem ∗ Considering special cases

Content and Learning Experiences

The content is divided into three strands viz. Algebra, Geometry and Trigonometry, and Calculus. In addition to the details for each content strand, the syllabus also includes learning experiences.

The learning experiences for N(A)-Level Additional Mathematics should provide opportunities for students to

- enhance conceptual understanding through use of various mathematical tools including ICT tools;
- make connections between representations, topics and methods;
- reason and communicate using mathematics language;
- apply mathematics knowledge and skills to real-world and other disciplines; and
- appreciate the beauty and value of mathematics.

Knowledge of the content of N(A)-Level Mathematics syllabus and the following additional topics are assumed.

1. Equations and Inequalities
   - Solving linear inequalities in one variable, and representing the solution on the number line

2. Functions and Graphs
   - Sketching of the graphs of quadratic functions given in the form
     - $y = (x-p)^2+q$
     - $y = -(x-p)^2+q$
     - $y = (x-a)(x-b)$
     - $y = -(x-a)(x-b)$

Content	Learning Experiences
ALGEBRA	**Students should have opportunities to:**
A1 Equations and inequalities	
1.1 Conditions for a quadratic equation to have: ∗ two real roots ∗ two equal roots ∗ no real roots and related conditions for a given line to: ∗ intersect a given curve ∗ be a tangent to a given curve ∗ not intersect a given curve 1.2 Conditions for ax^2+bx+c to be always positive (or always negative)	(a) Explain how the roots of the equation $ax^2+bx+c=0$ are related to the sign of b^2-4ac. (b) Transform ax^2+bx+c to the form $a(x-h)^2+k$ and use it to (i) sketch the graph; and (ii) deduce the quadratic formula. (c) Use a graphing software to investigate how the positions of the graph $y=ax^2+bx+c$ vary with the sign of b^2-4ac, and describe the graph when b^2-4ac < 0. (d) Use a graphing software to investigate the relationship between the number of points of intersection and the nature of solutions of a pair of simultaneous equations, one linear and one quadratic.

Content	Learning Experiences
ALGEBRA	**Students should have opportunities to:**
1.3 Solving simultaneous equations in two variables with at least one linear equation, by substitution 1.4 Relationships between the roots and coefficients of a quadratic equation 1.5 Solving quadratic inequalities, and representing the solution on the number line	(e) Examine the solution of a quadratic equation and that of its related quadratic inequality (e.g. $4x^2+x-5=0$ and $4x^2+x-5>0$), and describe both solutions and their relationship.
A2 Indices and surds	
2.1 Four operations on indices and surds, including rationalising the denominator 2.2 Solving equations involving indices and surds	(a) Make sense of numbers in surd form and recognise that the quadratic formula gives the real roots of quadratic equations in various forms (integer, rational number and conjugate surds).
A3 Polynomials	
3.1 Multiplication and division of polynomials 3.2 Use of remainder and factor theorems 3.3 Factorisation of polynomials	(a) Make connections between division of polynomial and division of whole number, and express the division algorithm as P(x) = (x – a)Q(x) + R. (b) Associate the remainder and factor theorems with the division algorithm. (c) Use a graphing software to investigate the graph of a cubic polynomial and discuss (i) the linear factors of the polynomial and the number of real roots; and

Content	Learning Experiences
ALGEBRA	**Students should have opportunities to:**
3.4 Use of: $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$ 3.5 Solving cubic equations	(ii) the number of real roots of the related cubic equation, with reference to the points of intersection with the x-axis.
A4 Binomial expansions	
4.1 Use of the Binomial Theorem for positive integer n 4.2 Use of the notations $n!$ and $\binom{n}{r}$ 4.3 Use of the general term $\binom{n}{r}a^{n-r}b^r$, $0 < r \leq n$ (knowledge of the greatest term and properties of the coefficients is not required)	(a) Expand $(a+b)^n$ $n=2,3,4,....$ and generalise the result to the binomial theorem. (b) Make connections between the coefficients of the binomial expansion and the Pascal Triangle, and use it to find the coefficients when n is small.

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
G1 Trigonometric functions, identities and equations	
1.1 Six trigonometric functions for angles of any magnitude (in degrees or radians) 1.2 Principal values of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ 1.3 Exact values of the trigonometric functions for special angles $(30°, 45°, 60°)$ or $(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3})$ 1.4 Amplitude, periodicity and symmetries related to sine and cosine functions 1.5 Graphs of $y = a\sin(bx) + c$, $y = a\sin(\frac{x}{b}) + c$, $y = a\cos(bx) + c$, $y = a\cos(\frac{x}{b}) + c$ and $y = a\tan(bx)$, where *a* is real, *b* is a positive integer and *c* is an integer. 1.6 Use of: * $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$,	(a) Discuss the relationships between $\sin A, \cos A$ and $\tan A$, , with respect to the line segments related to a unit circle. (b) Use a graphing software to display the graphs of trigonometric functions and discuss their behaviours, and investigate how a graph (e.g. $y = a\sin bx + c$) changes when a, b or c varies. (c) Relate the sine and cosine functions to sciences (e.g. tides, Ferris wheel and sound waves). (d) Relate $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$ to the sine, cosine and tangent functions respectively (e.g. $\sin^{-1}x$ is an angle whose sine is x, and the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is -30° or $-\frac{\pi}{6}$). (e) Deduce the formula for sin (A+B) from the sine rule.

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
$\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\text{cosec}^2 A = 1 + \cot^2 A$ ∗ the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ ∗ the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ ∗ the expression of $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$ 1.7 Simplification of trigonometric expressions 1.8 Solution of simple trigonometric equations in a given interval (excluding general solution) 1.9 Proofs of simple trigonometric identities	
G2 Coordinate geometry in two dimensions	
2.1 Condition for two lines to be parallel or perpendicular 2.2 Midpoint of line segment 2.3 Area of rectilinear figure	(a) Relate the gradient of a straight line to the tangent of the angle between the line and the positive direction of the x-axis, and deduce the relationship between the gradients of (i) two parallel lines and (ii) two perpendicular lines. (b) Discuss how to solve geometry problems involving finding (i) the equation of a line perpendicular or parallel to a given line, (ii) the coordinates of the midpoint of a line segment (horizontal, vertical and oblique), and (iii) equation of the perpendicular bisector of a line segment.

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
	(c) Explore and discuss ways of finding the area of a triangle (or polygon) with given vertices.
2.4 Graphs of parabolas with equations in the form $y^2 = kx$ 2.5 Coordinate geometry of circles in the form: • $(x-a)^2 + (y-b)^2 = r^2$ • $x^2 + y^2 + 2gx + 2fy + c = 0$ (excluding problems involving two circles)	(d) Use a graphing software to investigate the graph of $y^2 = kx$ when k varies. (e) Relate parabolas to examples in sciences and in the real world. (f) Make connections between the graphs of $y^2 = x$ and $y = x^2$. (g) Derive the equation of a circle with centre (a, b) and radius r using the Pythagoras theorem, and the special case when the centre is at the origin. (h) Discuss how to solve geometry problems involving intersection of a parabola/circle and a straight line.

Content	Learning Experiences
CALCULUS	**Students should have opportunities to:**
C1 Differentiation and integration	
1.1 Derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a point 1.2 Derivative as rate of change 1.3 Use of standard notations $\mathrm{f}'(x), \mathrm{f}''(x), \frac{dy}{dx}$, $\frac{d^2y}{dx^2} [= \frac{d}{dx}(\frac{dy}{dx})]$ 1.4 Derivatives of x^n, for any rational n, together with constant multiples, sums and differences 1.5 Derivatives of products and quotients of functions 1.6 Derivatives of composite functions 1.7 Increasing and decreasing functions 1.8 Stationary points (maximum and minimum turning points and stationary points of inflexion) 1.9 Use of second derivative test to	(a) Relate the derivative of a function to the gradient of the tangent to a curve at a given point, including horizontal and vertical tangents. (b) Distinguish between constant, average and instantaneous rate of change with reference to graphs. (c) Relate the sign of the first derivative of a function to the behaviour of the function (increasing or decreasing), locate points on the graph where the derivative is zero, and describe the behaviour of the function before, at and after these points. (d) Discuss cases where the second derivative test to discriminate between maxima and minima fails (e.g. $y = x^3$, $y = x^4$) and instead, use the first derivative test. (e) Discuss examples of problems in real-world contexts (e.g. business and sciences), involving the use of differentiation. (f) Explain what $\frac{d}{dx}(f(x))$, $\int f(x)dx$ and $\int_a^b f(x)dx$ represent and make connections between • derivative and indefinite integral; • definite and indefinite integrals. (g) Relate the area bounded by a curve and the y-axis to the area under the curve.

Content		Learning Experiences
CALCULUS		**Students should have opportunities to:**
	discriminate between maxima and minima	
1.10	Apply differentiation to gradients, tangents and normals, connected rates of change and maxima and minima problems	
1.11	Integration as the reverse of differentiation	
1.12	Integration of x^n for any rational n, (excluding $n=-1$), together with constant multiples, sums and differences	
1.13	Integration of $(ax+b)^n$ for any rational n, (excluding $n=-1$)	
1.14	Definite integral as area under a curve	
1.15	Evaluation of definite integrals	
1.16	Finding the area of a region bounded by a curve and line(s) (excluding area of region between 2 curves)	